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THE ADAPTIVE DETECTION AND ESTIMATION OF NEARLY PERIODIC SIGNALS

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Schenectady, New York

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TECHNICAL REPORT

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TO

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ABSTRACT

This report proposes a design of an adaptive receiver for the detection and estimation of nearly periodic signals in additive Gaussian noise. A nearly periodic signal is defined to be a sample function of a Gaussian random process which can be divided into equal length intervals, called periods, in such a manner that the correlation between periods decreases exponentially with their separation. The receiver computes a low signal-to-noise ratio conditional likelihood ratio from which the observer must make decisions. The likelihood ratio is conditional because the receiver estimates any unknown parameters necessary for the computation of the true likelihood ratio. Thus the receiver can only compute a likelihood ratio conditioned upon these estimates being the true values of the unknown parameters. The receiver consists of pre-emphasis filters followed by a comb filter, energy detector, and weighted summation. A theoretical evaluation of the receiver, in terms of ROC curves, is made for the special case of nearly periodic signals with satistically independent equal-strength harmonics in white noise of known power.

CONTENTS

THE ADAPTIVE DETECTION AND ESTIMATION OF NEARLY PERIODIC SIGNALS

T.G. Kincaid*

I. INTRODUCTION

This report proposes a system design of an adaptive receiver for the detection and estimation of nearly periodic signals in additive Gaussian noise. A nearly periodic signal is very much like a periodic signal, but fails to be periodic because its waveform is slowly changing with time. In this report, a nearly periodic signal is more precisely defined as a sample function of a Gaussian random process which can be divided into equal-length intervals, called periods, in such a manner that the correlation between periods decreases exponentially with their separation.

The receiver design is based on the philosophy of minimization of average decision cost, which leads to a receiver that computes a likelihood ratio and compares it with a threshold. However, the proposed receiver does not perform strictly in this manner for three reasons. First, since the parameters which determine the threshold are not known, the proposed receiver leaves the threshold comparison to an observer. Second, some parameters necessary for the design are estimated from the received data and inserted in place of the true values, which are unknown to the designer. Thus the receiver is only capable of computing a likelihood ratio conditioned upon the unknown parameters having their estimated values. Third, the proposed receiver is the low signal-to-noise ratio approximation to the receiver dictated by the theoretical analysis. This approximation results in comparatively simple receiver implementation without sacrificing threshold performance.

The operation of the proposed receiver can be satisfyingly interpreted in terms of the estimator-correlator structure described by Kailath,[24] followed by an incoherent summing operation. However, the receiver is more easily implemented if given an alternate interpretation, consisting of three distinct filtering operations followed by energy detection and the incoherent summing. The first two filtering operations are pre-emphasis filters which build up the high signal-to-noise ratio regions of the input. The third filter is a frequency domain comb filter with tooth separation equal to the inverse of the period of the nearly periodic signal, and tooth width determined by the intra period correlation of the nearly periodic signal. This comb filter is the heart of the receiver, and can easily be implemented as a circulating adder, which coherently adds weighted sequential periods of the pre-emphasized input. The comb filtering is followed by energy detection and an incoherent summing operation.

A partial evaluation of the receiver has been made by considering only the performance of the processor without the incoherent summing. Confining the evaluation to this part of the processor makes theoretical evaluation tractable. The evaluation is made for an input consisting of a nearly periodic signal with statistically independent constant amplitude harmonics and additive white Gaussian noise of known power. The evaluation is made in terms of Receiver Operating Characteristics (ROC), which display the probability of detection vs the probability of false alarm.

II. ADAPTIVE RECEIVER DESIGN PHILOSOPHY

This section sets forth the philosophy used to design the adaptive receiver for nearly periodic signals. The philosophy presented here is a summary of the literature in the area, and is general enough to be applied to a variety of adaptive receiver design problems.

A. Receiver Design

The basic function of any receiver is to make decisions about the values of one set of random variables (messages) given the values of a different set of random variables (data). We usually distinguish two different functions of receivers: detectors make decisions which are either right or wrong, and estimators make decisions which are seldom exactly right, but can often be close. The same receiver may perform both functions.

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How do you design such a receiver? As a starting point, we note that all we can know about the message random variables is contained in their joint probability density; thus, it seems reasonable that the receiver should first compute the joint probability density of the message variables conditioned upon the known values of the data variables. This is called the posterior probability density of the message; i.e., the density after the data are known. The receiver can then make decisions based on the computed posterior density, according to any criterion of goodness based on average performance. Our guiding principle will therefore be that the receiver is to be designed in two parts: (1) a posterior probability computer, and (2) an estimation or detection decision maker.

B. Adaptive Receivers

There seems to be no rigorous definition of what constitutes an adaptive system. Scudder's concept[2] of an adaptive system as one "--whose behavior changes with time, depending upon the input," is certainly broad enough to contain the thinking of most authors on the subject. However, this definition is too general to allow the design of a canonic adaptive receiving system, so the design is still an art.

A number of authors have shown how the use of the Bayes rule (Section II-C below) to compute posterior probabilities can lead to an adaptive processing system. This is accomplished either directly using a sequential form of Bayes' rule,[3-8] or indirectly by appropriate interpretation of the formulas describing the receiver resulting from the application of Bayes' rule.[9] However, the practical application of the Bayes rule approach has proved difficult. One problem is that the resulting receiver may grow exponentially in complexity as more and more data are received.[8] A more fundamental difficulty is the so-called "a priori" problem.[10-13] The a priori problem arises because it is necessary for the computation of posterior probabilities by Bayes' rule that the prior probabilities be known for both the message random variables and any unknown parameters required to design the receiver.

Some designers have avoided both the complexity and a priori problems by designing systems according to some criterion other than the computation of posterior probabilities.[14-16] For those who insist on the rigorous computation of posterior probabilities, Spragins[17] has demonstrated that the complexity problem can be overcome only by the use of sufficient statistics and reproducing densities, an area also investigated by Birdsall.[18] It appears that for many applications sufficient statistics can be found, although it may be necessary to make some approximations.[19] The more difficult a priori problem has not yet been satisfactorily resolved. The currently favored approach is to use the principle of maximum entropy, with any prior knowledge as constraints, to derive prior probability densities.[12, 19] If enough input data is available to the processor, it can be shown that under rather general conditions the choice of prior probability density is irrelevant, provided it does not exclude the true value of the parameter sought.[28]

Another approach (Section II-D below) to overcoming the complexity and a priori problems inherent in the use of Bayes' rule is to estimate the unknown parameters from received data, and then to insert the estimates in place of the actual values.[20, 21] This results in an inherently adaptive receiver, but one which does not actually compute the posterior probabilities. However, the use of estimates for the unknown parameters makes the receiver asymptotically equivalent to the desired posterior probability computing receiver if the estimates approach the true values. Even so, this limiting condition is never reached in practice, and often cannot even be approached arbitrarily closely because the true values of the parameters are changing. What this type of "suboptimal" receiver actually computes is the posterior probabilities of the desired variables, conditioned upon the unknown parameters having their estimated values.

In summary, an adaptive receiver can arise from at least two considerations, both of which may be used in the same receiver. The first is the use of the sequential form of Bayes' rule to compute posterior probabilities. The second is the use of current estimates of parameters in place of true values.

C. Computation of Posterior Probabilities

Suppose a receiver is given a data signal Z which contains information about a message random variable, Q, whose value we wish to determine. (Two random variables contain information about each other if they are dependent.) Then, according to our philosophy, the receiver first computes $f(Q/Z)$, the probability density function of Q given Z. The computation of $f(Q/Z)$ can be carried out using Bayes' rule, viz.

$$f(Q/Z) = \ell(Z/Q)f(Q)/f(Z) . \qquad \text{(II-1)}$$

The term $f(Q)$ is the prior probability density function of the message Q, i.e., the probability density function of Q before Z is received. As discussed above, the precise specification of this function is still a matter of philosophical discussion, and it figures prominently in the subsequent decisions. It is sufficient for now to note that the prior density does not depend upon the input Z.

The term $\ell(Z/Q)$ does depend upon Z. Although $\ell(Z/Q)$ is a probability density function for the random variable Z, we are interested in $\ell(Z/Q)$ as a function of Q, since we ultimately wish to compute the probability density function $f(Q/Z)$ on the left of Eq. (II-1). As a function of Q, $\ell(Z/Q)$ is not a probability density function, and it is commonly called the likelihood function. Note that since the likelihood function contains the input signal Z as a parameter, its functional form is dependent upon the input.

The term $f(Z)$, the prior probability density function of Z, is also dependent upon Z. However, it is not independent of the form of the likelihood function because of the constraint

$$\int dQ f(Q/Z) = \int dQ \, \ell(Z/Q)f(Q)/f(Z) = 1 . \tag{II-2}$$

Thus we can view $f(Z)$ as a normalizing constant, and write

$$f(Q|Z) = a \, \ell(Z|Q)f(Q) . \tag{II-3}$$

where

$$a^{-1} = \int dQ \, \ell(Z|Q)f(Q) . \tag{II-4}$$

Since $f(Q)$ is specified a priori, and since the normalizing constant a depends upon the likelihood function through Eq. (II-4), an important function of the receiver is the computation of the likelihood function $\ell(Z|Q)$.

We wish now to consider what happens when the receiver receives two successive inputs, Z_1 and Z_2, which contain information about the message Q. After the first input, the receiver computes

$$f(Q|Z_1) = a_1\ell(Z_1|Q)f(Q) . \tag{II-5}$$

After receiving the second input, the receiver computes

$$f(Q|Z_1, Z_2) = a_2\ell(Z_1, Z_2|Q)f(Q) , \tag{II-6}$$

in order to take advantage of all the information in both Z_1 and Z_2.

In general, for a set of k inputs $\{Z_i\}_k$ containing information about a message Q, the receiver computes

$$f(Q|\{Z_i\}_k) = a_k\ell(\{Z_i\}_k|Q)f(Q) , \tag{II-7}$$

with a_k computed as in Eq. (II-4). In Eq. (II-5), the likelihood function depends only on the input Z_1, and we have a "one-shot" receiver. In Eq. (II-7), the likelihood function depends upon the whole input sequence $\{Z_i\}_k$, and therefore describes a "multishot", or sequential, receiver. As with the "one-shot" receiver, an important function of the sequential receiver is the computation of the likelihood function $\ell(\{Z_i\}_k|Q)$.

As has already been noted, the likelihood function is a function of Q and not the $\{Z_i\}_k$ in the context of Eq. (II-7). Rather, the $\{Z_i\}_k$ are parameters of the likelihood function that determine its functional form. Thus the likelihood function will change in a manner determined by the receiver inputs. It is this view of the sequential computation of posterior probabilities which leads some authors to call this type of receiver adaptive.[9]

D. Treatment of Unknown Parameters

It almost always happens that the likelihood function contains parameters which are unknown, but which are critical to the evaluation of the likelihood function. These parameters may be associated with the message bearing signal (e.g., amplitude, phase), the noise (e.g., noise variance), or some other aspect of the problem. We shall describe two methods of dealing with this problem: the marginal density method, and the estimation method.

1. The Marginal Density Method

The most widely proposed[13, 22] method of handling unknown parameters is to compute the joint density of the message and the unknown parameters, and then to integrate over the unknown parameter random variables to obtain the (marginal) density of the message.

The computation is as follows. For a message Q and an unknown parameter A, the posterior density of Q is

$$f(Q|\{Z_i\}_k) = \int dA f(Q,A|\{Z_i\}_k)$$

$$= \int dA a_k \ell(\{Z_i\}_k|Q,A)f(Q,A)$$

$$= a_k[\int dA \ell(\{Z_i\}_k|Q,A)f(A|Q)]f(Q) . \quad \text{(II-8)}$$

Thus the likelihood function is

$$\ell(\{Z_i\}_k|Q) = \int dA \ell(\{Z_i\}_k|Q,A)f(A|Q) . \quad \text{(II-9)}$$

It often happens that Q and A are independent, in which case only the unconditional prior density of A is needed for the likelihood function.

The problem with this approach is that the likelihod function cannot be computed without the prior probability density of the unknown parameter. Since this density is usually not known, it must either be assumed or plausibly derived by some technique such as maximum entropy.

2. The Estimation Method

When the prior probability density of an unknown parameter is not known, a "suboptimal" receiver must be used. One possibility is to design the receiver as though the parameters were known, and then substitute estimates of the parameters for their true values.

To understand the meaning of the estimation method, consider the computation of the joint density of the message and the unknown parameter; however, instead of computing the complete joint density of the message Q and the unknown parameter A, we compute the probability density of Q only for A equal to its estimated value $\hat{A}$. Thus we compute

$$f(Q,A=\hat{A}|\{Z_i\}_k) = a_k \ell(\{Z_i\}_k|Q,A=\hat{A})f(Q,A=\hat{A}) . \quad \text{(II-10)}$$

After normalization, the function on the left of Eq. (II-10) becomes the density of Q given $\{Z_i\}_k$ and $A=\hat{A}$. Thus, the estimation method can be thought of as the computation of

$$f(Q|\{Z_i\}_k,\hat{A}) = a_k' \ell'(\{Z_i\}_k|Q,\hat{A})f(Q|\hat{A}) \quad \text{(II-11)}$$

where

$$a_k' = a_k \frac{\int dQ f(Q,A=\hat{A})}{\int dQ f(Q,A=\hat{A}|\{Z_i\}_k)} . \quad \text{(II-12)}$$

The constant a_k' is required to normalize the two probability density functions. Equation (II-11) shows that the resulting receiver computes the posterior density of the message, given that the parameter has its estimated value. This density converges to the desired posterior density of the message as the parameter estimate converges to its true value. For some kinds of estimates (Section II-E-2 below) it is not necessary to know the prior density of the unknown parameter. When these estimates are used, the a priori problem does not arise.

If the unknown parameter estimates are continually updated as new data are received, then the receiver will be constantly changing. Furthermore, the receiver may have the ability to change its form in response to a slowly changing parameter, e.g., noise variance. Thus a receiver which handles unknown parameters by the estimation method can be called adaptive.

E. Decision Making

1. Detection

We shall concern ourselves only with the problem of deciding between two possible values of a message variable, given the received data; thus, a message Q can take on only two values, Q_1 or Q_0. These might represent "target present" and "target absent," respectively. This restriction on Q gives the posterior density for Q a particular form.

$$f(Q|Z) = a\,\ell(Z|Q)[\mu\,\delta(Q-Q_1) + (1-\mu)\,\delta(Q-Q_0)]$$

$$= a\mu\,\ell(Z|Q_1)\,\delta(Q-Q_1) + a(1-\mu)\,\ell(Z|Q_0)\,\delta(Q-Q_0) \qquad \text{(II-13)}$$

where μ and $1-\mu$ are the prior probabilities of Q_1 and Q_0, respectively. Note that the posterior probabilities of Q_1 and Q_0 are $a\mu\,\ell(Z|Q_1)$ and $a(1-\mu)\,\ell(Z|Q_0)$, respectively.

Suppose we are interested in determining if $Q = Q_1$. Then the possible outcomes of the decision process are

1. detection - decide $Q = Q_1$ when $Q = Q_1$
2. miss - decide $Q = Q_0$ when $Q = Q_1$ (II-14)
3. false alarm - decide $Q = Q_1$ when $Q = Q_0$
4. (no name) - decide $Q = Q_0$ when $Q = Q_0$.

Of these four possibilities, two are correct decisions, and two are errors. The problem is to make the decisions in some best way.

We shall select as our ideal decision criterion the minimization of the expected cost of our decisions. We can denote the costs of the four outcomes as c_{ij}, where i refers to the decision and j to the actual value of Q. Thus c_{10} is the cost of deciding Q_1 when actually $Q = Q_0$. Then the cost, c, of any particular trial of the experiment is a random variable, which can take on any of the values c_{ij}.

We assume that the cost of a wrong decision is higher than the cost of a correct decision, i.e.,

$$c_{10} > c_{00} \qquad \text{(II-15)}$$

$$c_{01} > c_{11} \ . \qquad \text{(II-16)}$$

The receiver which minimizes the average cost $\langle c \rangle$ performs the Bayes test,[33, 34] viz.

$$\text{compare} \quad \frac{\mu\ell(Z|Q_1)}{(1-\mu)\,\ell(Z|Q_0)} \quad \text{with} \quad \frac{c_{10} - c_{00}}{c_{01} - c_{11}} \qquad \text{(II-17)}$$

and decides Q_1 if the ratio exceeds the threshold, Q_0 if not. The Bayes test needs posterior probabilities, and makes their role in the decision process explicit. It is more common to write the Bayes test in the form

$$\text{compare}\ \lambda(Z) = \frac{\ell(Z|Q_1)}{\ell(Z|Q_0)} \quad \text{with} \quad \frac{(1-\mu)(c_{10}-c_{00})}{\mu(c_{01} - c_{11})} \ . \qquad \text{(II-18)}$$

The quantity $\lambda(Z)$ is called the likelihood ratio. The likelihood ratio depends only upon the likelihood function $\ell(Z|Q)$, and is independent of the prior probabilities of Q. One major difficulty with the Bayes test is determining the threshold setting on the right of relation (II-18). Not only is there the a priori problem, but there is the additional problem of assigning costs. It seems the best that can be done is to present the observer with the likelihood ratio, and let him make decisions. In effect, the observer inserts the prior probabilities and costs from his experience and knowledge of the tactical situation. From this combination he sets a mental threshold and makes his decision.

A second problem with the Bayes test is that the likelihood ratio cannot be computed exactly if the prior probabilities of the unknown parameters are not available. Our philosophy then is to use the conditional posterior probabilities determined by the estimation method described in Section II-D-2;that is, estimates of the parameters are used as though they were the true values. This results in a conditional likelihood ratio (author's definition). For an unknown parameter with estimate $\hat{A}$, the conditional likelihood ratio is

$$\lambda(Z|\hat{A}) = \frac{\ell(Z|Q_1,\hat{A})}{\ell(Z|Q_0,\hat{A})} \ . \qquad \text{(II-19)}$$

The methods by which the estimates can be made are described in the next section. When $\hat{A}$ is a maximum likelihood estimate (Section II-E-2 below) the conditional likelihood ratio is equivalent to the generalized likelihood ratio.(34)

Since it may not be possible to evaluate the receiver by costs, we need another method of evaluating the receiver. A commonly used method that is independent of the message prior probabilities is the receiver operating characteristic (ROC). This is the plot of the probability of a detection (p_d) vs the probability of a false alarm (p_f) when the receiver compares the likelihood ratio with a threshold level η. It has been shown by Neyman and Pearson(34) that a receiver which makes detections by comparing the actual likelihood ratio with a threshold will maximize the probability of detections for a fixed probability of false alarms. This theorem ensures that the ROC curves will rate the receiver which computes the true likelihood ratio superior to the one which computes a conditional likelihood ratio.

The ROC curves are determined as follows. The (conditional) likelihood ratio, λ, is a random variable. The probability of detection is given by

$$p_d = \int_\eta^\infty d\lambda \; f(\lambda \mid Q_1) \tag{II-20}$$

and the probability of a false alarm is given by

$$p_f = \int_\eta^\infty d\lambda \; f(\lambda \mid Q_0) \; . \tag{II-21}$$

Then, the ROC curves are computed by choosing values of η, computing p_d and p_f, and plotting one vs the other.

2. Estimation

There are two principal reasons why we may need to make an estimate of the value of a random variable. It may be that the random variable is an unknown parameter and we wish to have the estimate for use in the estimation method of receiver design described in Section II-D-2. The second reason is that the random variable may be a message, and having an estimate of its value is important for reasons other than receiver design, e.g., target classification.

Whatever the reason, there are a number of different ways of making estimates, each having particular advantages in certain situations.

Unconditional estimates are based on both the received data and the prior density of the unknown parameter. Two widely used unconditional estimates are:

The Minimum Mean-Square Error (mms) Estimate. The mms estimate of a random variable, A, given the value of a random variable, Z, is the value $\hat{A}$ of A which minimizes

$$e^2 = \int dA \; (A - \hat{A})^2 \, f(A \mid Z) \; . \tag{II-22}$$

It is straightforward(34) to show that the minimum is given by the conditional mean of A.

$$\hat{A} = \int dA \; A f(A \mid Z) \; . \tag{II-23}$$

The Maximum Posterior Probability (mpp) Estimate. The mpp estimate of a random variable, A, given the value of a random variable Z, is the value $\hat{A}$ of A for which the posterior density of A is a maximum; in other words, $\hat{A}$ is the most probable value of A when Z is known.

$$\hat{A} = A \text{ for which } f(A \mid Z) \text{ is maximum.} \tag{II-24}$$

This estimate is also called the maximum *a posteriori* (map) estimate.

Conditional estimates depend only on the received data. Because of this, conditional estimates can be used in the estimation method of handling unknown parameters (Section II-D-2). Two popular conditional estimates are:

The Maximum Likelihood ($m\ell$) Estimate. The $m\ell$ estimate of a random variable, A, given the value of a random variable, Z, is the value $\hat{A}$ of A for which the likelihood function for A is a maximum.

$$\hat{A} = A \text{ for which } \ell(Z|A) \text{ is maximum.} \tag{II-25}$$

In other words, A is the value of A that most likely caused the given value of Z to occur.

The Likelihood Ratio (ℓr) Estimate. The ℓr estimate of a random variable, A, given a random variable, Z, is the value of $\hat{A}$ of A for which the likelihood ratio of a message Q is a maximum.

$$\hat{A} = A \text{ for which } \lambda(Z|A) \text{ is a maximum.} \tag{II-26}$$

This estimate is the most widely used for the estimation of unknown target parameters in detection problems; e.g., range in sonar. When used for this purpose, it is equivalent to the $m\ell$ estimate, given the target is present. The estimate is made by forming the likelihood ratio for all possible values of the unknown parameter, and choosing the largest ratio. This is, in effect, constructing a separate likelihood ratio detector for each value of the unknown parameter, and choosing the value of the parameter corresponding to the detector with the largest output.

III. ADAPTIVE DETECTION AND ESTIMATION OF NEARLY PERIODIC SIGNALS

The objective of this section is to design an adaptive receiver for the detection and estimation of a nearly periodic signal in additive, zero mean, Gaussian noise. The detection problem will be given primary consideration, the estimation coming as a by-product.

In this work it will be assumed that the nearly periodic signal is a sample function of a nonstationary Gaussian random process, of the type described in the Introduction. A more detailed description of the properties of the nearly periodic signal are given in Sections III-A, B below.

A. Known Parameters

We shall first solve the detection problem when all parameters are known except whether the signal is present or absent. Then, after the appropriate receiver is derived, the extension to unknown parameters will be made by the methods described in Section II-D.

We shall represent signals by column matrices whose entries are the time samples of the signals. We assume that the receiver input signal is either the noise only, or a sum of the noise and a nearly periodic signal with the properties described above. We further assume that only s successive time samples of the input are available. Then the matrix equation for the input signal is

$$Z = \theta X + N, \tag{III-1}$$

where Z is the input signal matrix;

θ is a random variable which is either 1 or 0, depending upon whether the nearly periodic signal is respectively present or absent;

X is the nearly periodic signal matrix, assumed known here; and

N is the zero mean Gaussian noise signal matrix.

For detection, the receiver computes the likelihood ratio

$$\lambda(Z) = \frac{\ell(Z|\theta=1)}{\ell(Z|\theta=0)} . \tag{III-2}$$

In the context of this problem, θ is the message Q in Eq. (II-1). Under our assumptions, the likelihood function in Eq. (III-2) can be written as

$$\ell(Z|\theta) = (2\pi)^{-s/2} |\Phi_N|^{-1/2} \exp\left\{- \frac{1}{2} (Z-\theta X)^* \Phi_N^{-1}(Z-\theta X)\right\} . \tag{III-3}$$

In this equation, Φ_N is the $s \times s$ noise covariance matrix, and * denotes matrix transposition.

We assume that when the input signal is sampled near the Nyquist rate for the nearly periodic signal, then there are exactly p time samples of the nearly periodic signal in each period. We further assume that the noise is significantly correlated only over a number of adjacent time samples which are small compared to p.

This implies that the noise spectrum does not vary too rapidly over the frequency band occupied by the nearly periodic signal.

Then we define Z_i to be a $1 \times p$ column matrix whose entries are the p entries of Z from $(i-1)p+1$ to ip inclusive, corresponding to one period of the periodic waveform, and X_i similarly. We will assume stationary noise and define Φ_{Np} to be the $p \times p$ matrix whose rows are the p entries of Φ_N contained in rows $(i-1)p+1$ to ip inclusive, and the columns of the same index, for any i.

Under the above assumption about the noise correlation, and using the notation described above, the likelihood function of Eq. (III-3) can be written approximately as

$$\ell(Z|\alpha) = (2\pi)^{-kp/2} |\Phi_{Np}|^{-k/2} \exp\left[-\frac{1}{2} \sum_{i=1}^{k} (Z_i - X_i)^* \Phi_{Np}^{-1} (Z_i - X_i)\right] \tag{III-4}$$

where k is the number of times p divides s. In general, p will not divide s evenly, and so Eq. (III-4) ignores a portion of the input signal in computing the likelihood function. However, for $k \gg 1$ (i.e., many periods), this excess signal will make a negligible contribution to the likelihood function, and so can be neglected for the sake of simplicity.

Equation (III-4) is an example of how the computation of the likelihood function for a specific problem can result in an adaptive interpretation for the processor. The equation shows that the likelihood function changes each time a new set of p samples is added to the input signal, giving a sequential type adaptive receiver as discussed in Section II-B.

B. Unknown Parameters

In general, Φ_{Np}, $\{X_i\}$, and p are unknown. In this case, the receiver can be designed using the methods described in Section II-D for handling unknown parameters. The methods may be intermixed. If possible, it is desirable to use the marginal density method, since it gives unconditional posterior probabilities.

For the set of unknown waveforms $\{X_i\}$, we shall use the marginal density method. We shall assume that the p samples in any period of the nearly periodic waveform are zero mean Gaussian random variables with covariance matrix Φ_{Xp}, which is the same for each period. We further assume that the samples in the ith and jth periods, which are $|i-j|$ periods apart, have a cross covariance matrix $\rho^{|i-j|} \Phi_{Xp}$, where $0 \le \rho \le 1$. From these definitions we see that the marginal density method is still going to leave the parameters Φ_{Xp} and ρ to be specified. We shall assume that these parameters are either known or will be estimated. In the latter case we still proceed as through these parameters were known, and substitute the estimated values for the true values at the end.

The parameters p and Φ_{Np} will be estimated. Since p is a parameter of a signal which may not be present, it will be estimated using the likelihood ratio estimate (Section II-E-2), i.e., by building parallel receivers, one for each value of p. The noise is always present, and its covariance can be estimated continuously by maximum likelihood or some other technique. We therefore proceed as though p and Φ_{Np} were known.

Now the likelihood function for α is

$$\begin{aligned} \ell(Z|\alpha) &= \int dX\, \ell(Z|\alpha, X)\, f(X|\alpha) \\ &= \int \{dX_i\}_k\, \ell(Z|\alpha, \{X_i\}_k)\, f(\{X_i\}_k) \;, \end{aligned} \tag{III-5}$$

assuming the $\{X_i\}_k$ are independent of α. The integral on the set $\{X_i\}_k$ is to be interpreted as the kp-fold integral on the entries of X. In Eq. (III-5), the first term of the integrand is given by Eq. (III-4). The second term is the prior density of the set of waveforms $\{X_i\}_k$, i.e., the prior density of X.

An expression for the prior density of the $\{X_i\}_k$ can be derived as follows. From the above definitions, the covariance matrix for the $s = kp$ samples from the periods of the nearly periodic waveform X is the $s \times s$ matrix Φ_X, which can be written as the Kronecker product[26, 27]

$$\Phi_X = P \otimes \Phi_{Xp} \tag{III-6}$$

where

$$P = \begin{bmatrix} 1 & \rho & \rho^2 & \cdots & \rho^{k-1} \\ \rho & 1 & \rho & \cdots & \vdots \\ \rho^2 & \rho & 1 & \cdots & \vdots \\ \vdots & & & & \vdots \\ \rho^{k-1} & \cdots & \cdots & \cdots & 1 \end{bmatrix} \qquad \text{(III-7)}$$

From the prope... s of Kronecker products, the inverse of Φ_X is

$$\Phi_X^{-1} = P^{-1} \times \Phi_{Xp}^{-1} \qquad \text{(III-8)}$$

where

$$P^{-1} = (1-\rho^2)^{-1} \begin{bmatrix} 1 & -\rho & 0 & \cdots & 0 \\ -\rho & 1+\rho^2 & -\rho & & \vdots \\ 0 & -\rho & 1+\rho^2 & & \vdots \\ \vdots & & & \ddots & \vdots \\ \vdots & & & 1+\rho^2 & \vdots \\ 0 & \cdots & \cdots & \cdots & 1 \end{bmatrix} \qquad \text{(III-9)}$$

This inverse of P can be verified by direct multiplication. Thus we can write

$$\begin{aligned} f(\{X_i\}_k) &= f(X) \\ &= (2\pi)^{-kp/2} |\Phi_X|^{-1/2} \exp\left\{-\frac{1}{2} X' \Phi_X^{-1} X\right\} \\ &= (2\pi)^{-kp/2} |P|^{-p/2} |\Phi_{Xp}|^{-k/2} \\ &\quad \exp\left\{-\frac{1}{2}(1-\rho^2)^{-1}\left[\sum_{i=1}^{k} X_i' \Phi_{Xp}^{-1} X_i + \rho^2 \sum_{i=2}^{k-1} X_i' \Phi_{Xp}^{-1} X_i \right.\right. \\ &\quad \left.\left. - \rho \sum_{i=1}^{k-1} X_{i+1}' \Phi_{Xp}^{-1} X_i - \rho \sum_{i=1}^{k} X_i' \Phi_{Xp}^{-1} X_i \right]\right\} \end{aligned} \qquad \text{(III-10)}$$

Now the likelihood function can be evaluated by substituting Eqs. (III-4) and (III-10) into Eq. (III-5) and integrating. The integral is conveniently evaluated separately for the two values of θ. Carrying out the integration, the likelihood function $\theta=0$ is

$$\ell(Z|\theta=0) = (2\pi)^{-kp/2} |\Phi_{Np}|^{-k/2} \exp\left\{-\frac{1}{2}\sum_{i=1}^{k} Z_i' \Phi_{Np}^{-1} Z_i\right\}, \qquad \text{(III-11)}$$

and the likelihood function for $\theta=1$ is shown in Appendix A to be

$$\begin{aligned} \ell(Z|\theta=1) &= (2\pi)^{-kp/2} |\Phi_{Np}|^{-k/2} \exp\left\{-\frac{1}{2}\sum_{i=1}^{k} Z_i' \Phi_{Np}^{-1} Z_i\right\} \\ &\quad |\Phi_{Xp}|^{-k/2} (1-\rho^2)^{-p(k-1)/2} \prod_{i=1}^{k} |A_i|^{1/2} \exp\left\{\frac{1}{2}\sum_{i=1}^{k} c_i\right\} \end{aligned} \qquad \text{(III-12)}$$

where A_i and c_i are given by Eqs. (A4) and (A6) in Appendix A.

Using these results, the likelihood ratio is

$$\lambda(Z) = \frac{\ell(Z|\theta=1)}{\ell(Z|\theta=0)}$$

$$= |\Phi_{Xp}|^{-k/2}(1-\rho^2)^{-p(k-1)/2} \prod_{i=1}^{k} |A_i|^{1/2} \exp\left\{\frac{1}{2}\sum_{i=1}^{k} c_i\right\}. \tag{III-13}$$

A more convenient parameter is the logarithm of the likelihood ratio.

$$\lambda(Z) = \ln\left[|\Phi_{Xp}|^{-k/2}(1-\rho^2)^{-p(k-1)/2} \prod_{i=1}^{k} |A_i|^{1/2}\right] + \frac{1}{2}\sum_{i=1}^{k} c_i \; . \tag{III-14}$$

Since the parameters p, Φ_{Np}, ρ, and Φ_{Xp} are to be estimated if they are not known, the above equations are really for a conditional likelihood ratio, as discussed in Section II-E-1.

IV. PRACTICAL RECEIVER DESIGN AND EVALUATION

A. Practical Receiver Design

1. General Structure

In the previous section, it was shown that the receiver for the detection of a nearly periodic signal computes the logarithm of the likelihood ratio given in Eq. (III-14). The expressions for the coefficients A_i and c_i given by Eqs. (A4 and A6) are complicated and do not lend themselves to a simple receiver design.

A simple receiver design results if the receiver computes the logarithm of the likelihood ratio only for low signal-to-noise ratios. This is also a practical constraint, since it is not necessary for the receiver to be sensitive at high signal-to-noise ratios where detection is much easier. We define low signal-to-noise ratio as the condition where the eigenvalues of the signal-to-noise ratio matrix, $\Phi_{Xp}\Phi_{Np}^{-1}$, are much less than unity.[21] Under this condition, it is shown in Appendix B that the coefficients A_i and c_i are approximated by Eqs. (B2) and (B3), respectively. Substituting these equations into Eq. (III-14), the logarithm of the likelihood ratio becomes

$$\begin{aligned}\lambda &= \ln\left[(1-\rho^2)^{-p/2}\right] \\ &+ \frac{1}{2}\left[(1-\rho^2)\sum_{i=1}^{k-1}\left(\sum_{j=1}^{i}\rho^{i-j}\Phi_{Np}^{-1}Z_j\right)^*\Phi_{Xp}\left(\sum_{j=1}^{i}\rho^{i-j}\Phi_{Np}^{-1}Z_j\right)\right] \\ &+ \frac{1}{2}\left[\left(\sum_{j=1}^{k}\rho^{k-j}\Phi_{Np}^{-1}Z_j\right)^*\Phi_{Xp}\left(\sum_{j=1}^{i}\rho^{k-j}\Phi_{Np}^{-1}Z_j\right)\right].\end{aligned} \tag{IV-1}$$

In Eq. (IV-1), the first term is an additive constant, unaffected by the input to the receiver. The last two terms are the ones which need to be computed by the receiver. We shall denote these terms, without the 1/2 multiplier, by

$$q = (1-\rho^2)\sum_{i=1}^{k-1} q_i + q_k \tag{IV-2}$$

where

$$q_i = \left(\sum_{j=1}^{i}\rho^{i-j}\Phi_{Np}^{-1}Z_j\right)^*\Phi_{Xp}\left(\sum_{j=1}^{i}\rho^{i-j}\Phi_{Np}^{-1}Z_j\right) \; ; \; i = 1 \text{ to } k \; . \tag{IV-3}$$

The heart of this low signal-to-noise ratio receiver is the computation of the q_i. By interpreting the square matrices Φ_{Xp} and Φ_{Np}^{-1} as linear filters[24, 35] (which may be time variable and/or unrealizable), this

computation can be given at least two interpretations: the estimator correlator interpretation, and the filter-energy detector interpretation. The estimator-correlator interpretation shows the connection between the results obtained here and the work of Kailath,[24] and also allows an adaptive explanation of the receiver operation which is a particular case of the work of Nolte,[9] and closely parallels the work of Jakowatz, Shuey, and White.[14] The filter-energy detector interpretation does not provide the same insights into the receiver operation but is simpler both in concept and implementation than the estimator-correlator interpretation.

The estimator-correlator interpretation comes about from the following manipulation of Eq. (IV-3). Since the matrix Φ_{Np}^{-1} is positive definite, it can be split into the product of a triangular matrix and its transpose, as follows:[32]

$$\Phi_{Np}^{-1} = W_p^* W_p \tag{IV-4}$$

then Eq. (IV-3) can be written

$$\begin{aligned} q_i &= \left(\sum_{\ell=1}^{i} \rho^{i-\ell} W_p Z_\ell\right)^* \left(W_p \sum_{j=1}^{i} \rho^{i-j} \Phi_{Xp} \Phi_{Np}^{-1} Z_j\right) \\ &= \sum_{\ell=1}^{i} \rho^{i-\ell} \left[\left(W_p Z_\ell\right)^* \left(W_p \sum_{j=1}^{i} \rho^{i-j} \Phi_{Xp} \Phi_{Np}^{-1} Z_j\right) \right] \end{aligned} \tag{IV-5}$$

Kailath[24] has shown that the filter W_p is a prewhitening filter, and in Eq. (IV-5) W_p prewhitens the noise component of each period Z_ℓ of the received signal. The filter cascade $\Phi_{Xp}\Phi_{Np}^{-1}$ is the low signal-to-noise ratio approximation to the Wiener filter which estimates the signal X_j from Z_j (see Kailath[24]). In Eq. (IV-5) these individual estimates are weighted by the ρ^{i-j} and added to form an over-all estimate,

$$\hat{X}_i = \sum_{j=1}^{i} \rho^{i-j} \Phi_{Xp} \Phi_{Np}^{-1} Z_j \ , \tag{IV-6}$$

of the current period X_i of the nearly periodic signal. This over-all estimate is distorted by the filter W_p to compensate for the distortion of Z_i by W_p, and then correlated with each of the i prewhitened input periods $W_p Z_\ell$. The matrix product inside square brackets in Eq. (IV-5) is this correlation. The scalars that result from the i correlations are then weighted by the $\rho^{i-\ell}$ and summed to give the q_i. This interpretation of the receiver can be given an adaptive character by viewing it as a correlation receiver in which the reference waveform $\hat{X}_i$, given by Eq. (IV-6), is not fixed but instead estimated from the received data. Thus the reference waveform adapts itself somewhat in the manner of the filter built by Jakowatz, Shuey, and White,[14] but following the theory described by Nolte.[9] It is of interest to note that when $\rho = 0$ Eq. (IV-5) reduces to

$$q_i = (W_p Z_i)^* (W_p \Phi_{Xp} \Phi_{Np}^{-1} Z_i) \ , \tag{IV-7}$$

which is the low signal-to-noise ratio approximation to Kailath's[24] zero mean signal receiver.

The filter-energy detector interpretation comes about as follows. Since the matrix Φ_{Xp} is positive definite, it can be split into the product of a triangular matrix and its transpose.[32]

$$\Phi_{Xp} = G^* G \ . \tag{IV-8}$$

Then Eq. (IV-3) can be written as the vector squared magnitude

$$q_i = \left\| \sum_{j=1}^{i} \rho^{i-j} G \Phi_{Np}^{-1} Z_j \right\|^2 \ . \tag{IV-9}$$

Equation (IV-9) shows that the q_i are the result of passing the Z_i through the filters G and Φ_{Np}^{-1}, weighting the filtered outputs by ρ^{i-j}, adding them together, and computing the energy of the sum. The total filter $G\Phi_{Np}^{-1}$ can be interpreted in terms of prewhitening and low signal-to-noise ratio Wiener filtering, but a straightforward interpretation in terms of pre-emphasis filtering suffices here. The filter Φ_{Np}^{-1} has the effect of emphasizing the regions of the spectrum where the noise power is low, and it emphasizes these regions more strongly than

a prewhitening filter. The filter G is the filter whose spectrum is the square root of the spectrum of Φ_{Xp}. This filter emphasizes the portion of the spectrum where the desired signal is expected to be, and de-emphasizes the other regions.

The summation which follows these two filters is the means by which the receiver takes advantage of the nearly periodic structure of the desired signal. The filtered waveform sections are weighted and added on top of each other, and when there is a nearly periodic waveform present, this sum will be larger than when there is not. This coherent summing operation is sometimes referred to as a circulating addition, because it can be implemented by a delay line with feedback, i.e., a recirculating line.

This circulating adder is completely equivalent to a frequency domain comb filter acting on the output of the G and Φ_{Np}^{-1} pre-emphasis filters. This equivalence is shown in Appendix C, with the frequency response Y(f) of the filter given by Eq.(C2). Taking the sampling interval as the fundamental time unit, the comb filter "tooth" separation is shown to be $1/p$, and the tooth width between the 1/2 power points is $(1/\pi p) \ln (1/\rho)$, as shown in Fig. 1.

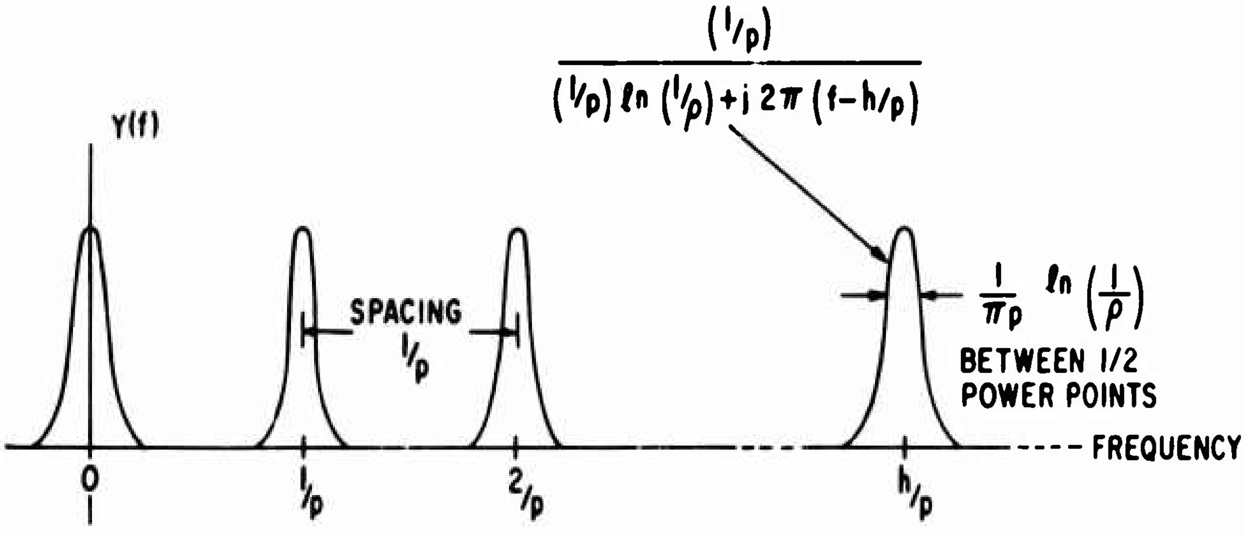

Fig. 1 The frequency response of the comb filter interpretation of the weighted circulating adder.

With this interpretation of the circulating adder as a linear time invariant filter, the portion of the receiver which computes q_i becomes rather simple. It is a cascade of the Φ_{Np}^{-1}, G, and comb filters, followed by an energy detector. The energy detector computes the energy of the filter output in each successive period, and these energies are the q_i. The remainder of the operation of the receiver is dictated by Eq.(IV-2). This equation shows that the quantity q, which determines Λ, is a weighted sum of the q_i. All the q_i but the last are weighted by $(1-\rho^2)$; this last q_k is given unit weight. This weighting means that the energy of the most recent p length section of the filtered input is given considerably more weight than the energies of previous sections. This is not surprising, since the most recent coherent summation embodied in the comb filtering operation has made use of practically the same information as the previous summations, except for the last p-length section. Thus, the contribution of the individual previous coherent summations is de-emphasized relative to the last. However, the contribution of the sum of these previous terms is not negligible for k on the order of $(1-\rho^2)^{-1}$ and larger. This incoherent processing can make an important contribution to the value of q.

The discussion of this section can now be summarized in a block diagram of the proposed receiver for nearly periodic signals. This block diagram is shown in Fig. 2. In the following sections, a discussion of the implementation of each block is given.

2. Details of Structure

The noise pre-emphasis filter structure depends upon the noise covariance matrix, which we have assumed is not known to the designer. We have proposed to estimate this matrix from the input data. The difficulty is that this matrix must be inverted to design the filter. This inversion is made relatively simple by the assumption

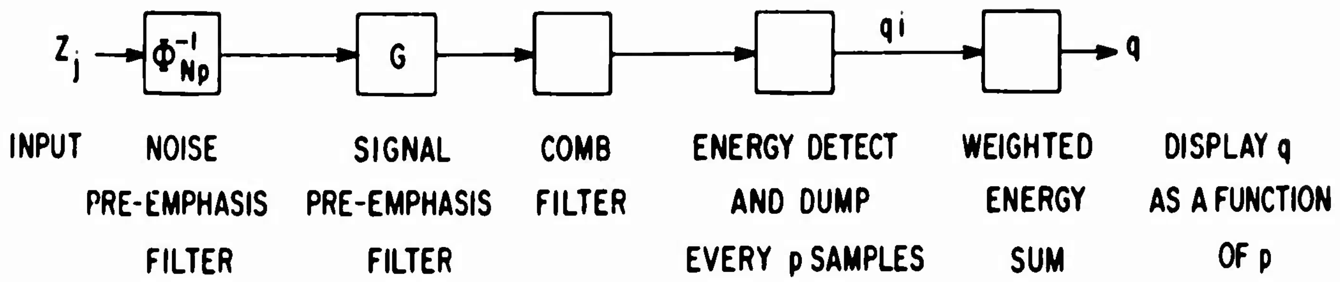

Fig. 2 Block diagram of the nearly periodic signal detector and estimator. One such detector is required for each period p.

that the noise is nearly uncorrelated over a whole period of the nearly periodic signal. For stationary noise, this means that the noise covariance matrix is of the form

$$\Phi_{Np} = \sigma^2_{Np} (I + E) \tag{IV-10}$$

where σ^2_{Np} is the noise variance, I is the identity matrix, and E is a matrix whose off-diagonal entries are much less than one, and whose diagonal entries are zero. The quantity of interest is the inverse of this matrix, which can be approximated by the first two terms of the binomial expansion for matrices

$$\begin{aligned} \Phi^{-1}_{Np} &= [\sigma^2_{Np} (I + E)]^{-1} \\ &\approx \sigma^{-2}_{Np} (I - E) . \end{aligned} \tag{IV-11}$$

The variance σ^2_{Np} and the coefficients of the E matrix can be estimated, for example, from the data by subtracting the estimate of the desired signal from the input and averaging the products of the remaining data at appropriate time delays.

The signal pre-emphasis filter G is uniquely determined from the nearly periodic signal covariance matrix Φ_{Xp} in the manner indicated by Eq. (IV-8). It is necessary either to know this matrix, or to estimate it. For a large number of possible Φ_{Xp}, the parallel receiver method is impractical. There seems to be no alternative to having as much knowledge of Φ_{Xp} as possible available to the designer. Simply stated, this means the strengths and cross correlations of the harmonics of the nearly periodic signal must be known.

It should be noted that both the noise pre-emphasis filter and the signal pre-emphasis filter are time varying. These filters must be realized by an operation equivalent to multiplying the samples of each individual period of the input by a square matrix, and taking the result as the samples of the corresponding period of the output. Furthermore, the noise pre-emphasis filter is unrealizable in the classical sense because the Φ^{-1}_{Np} matrix is not triangular. The time variant and unrealizable features of these filters prevent their easy implementation by analog networks. However, the matrix multiplications can be easily performed by a computer, with speed as the main limitation.

The comb filter, or weighted circulating adder, is the heart of the processor. The circulating adder is easy to implement, especially with a computer. The only parameters of this filter are p and ρ. The parameter p is to be handled by the estimation method, and hence need not be known, since all values of interest can be tried using parallel receivers. The parameter ρ must either be known, or estimated in the same manner as p. The estimation of these parameters is discussed further below.

The energy detection and weighted energy sum portions of the processor are fairly straightforward. The energy detector simply computes the energy q_i in each period of the comb filter output, and outputs this information every period. The weighted sum q is then found by adding all the q_i weighted by $1-\rho^2$, except the most recent, which is weighted by unity. This raises the question of how many q_i should be added. This question has not arisen before because it is inherent in the formulation of the problem that the nearly periodic

signal is either present or absent with fixed period for the whole experiment, so the q_i can be added indefinitely. If this is not true, the q_i can be added only as long as the nearly periodic signal is present with a fixed period, otherwise the performance is degraded. It seems that this decision must be made by the designer. It may even be advantageous to leave out the weighted energy sum altogether, substituting eye integration in connection with an appropriate display.

3. Parameter Estimates

As indicated at the beginning of Section III, the estimation problem is automatically solved as a by-product of the detection problem. The period p is to be estimated as part of the detection problem, in the manner described below in Section IV-A-3. The parameter ρ can be estimated also, but need not actually be displayed. The receiver can compute the likelihood function for several values of ρ, and always choose to display the output which gives the largest value of q.

Since it is impossible to compute the likelihood ratio for every value of p and ρ, it is of interest to know how far apart p and ρ can be sampled without missing a peak in the likelihood ratio. In Appendix C it is shown that the period p should be sampled at intervals not greater than

$$\Delta p = \frac{p}{\pi h_{max}} \ln \left(\frac{1}{\rho}\right) \tag{IV-12}$$

where h_{max} is the number of the highest nonzero amplitude harmonic in the nearly periodic signal. Similarly, it is shown in Appendix C that the correlation coefficient ρ should be sampled at an interval not greater than

$$\Delta\rho = \rho(1-\rho) \; . \tag{IV-13}$$

The remaining estimate of interest is the waveform of one period of the nearly periodic waveform. Since this parameter is changing from one period to the next, no special emphasis is placed on the estimation of this parameter. Previous work on the estimation of periodic signals[30] indicates that a reasonable waveform estimate can be obtained at the output of the comb filter if the pre-emphasis filters are removed.

No analysis of the goodness of the parameter estimates has been attempted here. Such an analysis is extremely difficult because of the use of the estimated noise covariance in the pre-emphasis filter, and is perhaps best left to a simulation. For an analysis of the estimation of periodic signals in noise, see Kincaid and Scudder.[30]

4. Display

The receiver output display would ideally be the value of q. In practice, a reasonable thing to do is to display q as a function of p, to give the observer a feeling for the output when there is no nearly periodic waveform present. This function should be displayed at successive time intervals to give a three-dimensional display having time as one of the dimensions, the other two being q and p. The observer then has (in effect) the conditional likelihood ratio as a function of p and time. From this he must estimate the period and make a decision.

The observer would not consciously go through the mathematics required to make the decision. Instead, he would look for peaks in the display which are significantly higher than the surrounding level. After some experience, the operator is usually able to tell if these bumps are significant, which is presumably similar to estimating the period, inserting the necessary a priori probabilities and costs, and making a decision.

B. Receiver Evaluation as a Detector

The theoretical evaluation of the receiver in terms of average cost is precluded by the fact that we cannot specify prior probabilities and decision costs. A standard measure of performance is the Receiver Operating Characteristics (ROC), described in Section II-E-1.

In our case, the output which we should test against a threshold to obtain ROC plots is the random variable q given by Eq. (IV-2). Unfortunately, it is in general very difficult to get the probability distribution of q when the signal is present. However, ROC curves can be obtained for a detector which computes just q_k instead of the sum q. This means that the detector does not perform the incoherent summing operation after filtering and energy detecting. This detector is easier to evaluate because it eliminates the problem of handling the sum of the highly dependent q_i when the signal is present.

In order to make an evaluation of the proposed receiver for a particular case, consider the following. Assume the nearly periodic signal in each period is of the form

$$X_p(t) = \mathrm{Re} \sum_{m=1}^{\ell} a_m \exp(j\alpha_m) \exp[j2\pi(m/p)t] \tag{IV-14}$$

where the a_m are independent random variables with the same known variance σ_X^2, and p is assumed known. The α_m are uniformly distributed between $-\pi$ and π, and are independent of the a_m. The period p, the period-to-period correlation ρ, and the number of components ℓ are assumed known. In general terms, Eq. (IV-14) is a model of a signal which is the sum of ℓ random harmonic "line" components, where the spectral width of the lines is determined by ρ. Each harmonic is independent of the others, but they all have the same average power. We assume the noise is white with known variance $\ell\sigma_N^2$.

Under these assumptions, it is shown in Appendix D that the random variable q_k is chi-squared distributed with 2ℓ degrees of freedom, both when the signal is present and when it is absent. The expected value of q_k is shown to be

$$\langle q_k \rangle = \begin{cases} \dfrac{2\ell r}{1-\rho^2}\left(1 + \dfrac{1+\rho^2}{1-\rho^2} r\right) & \text{for } \theta=1 \\ \dfrac{2\ell r}{1-\rho^2} & \text{for } \theta=0 \end{cases} \tag{IV-15}$$

where $r = \sigma_X^2 \sigma_N^{-2}$, i.e., the input signal-to-noise ratio. We could define the ratio of these two expectations as the output signal-to-noise ratio

$$\mathrm{snr} = 1 + \frac{1+\rho^2}{1-\rho^2} r \ . \tag{IV-16}$$

By this definition the snr is independent of the number of harmonics ℓ. However, the ROC curves for this detector shown in Fig. 3 reveal a strong dependence upon ℓ. The computation of these curves explained in Appendix D. Note that the curves are dependent upon the product nr, where n is the number of periods at which the signal correlation drops to 50%, and r is the input signal-to-noise ratio. The quantities n and ρ are related by

$$\rho^n = \frac{1}{2} \ . \tag{IV-17}$$

The effect of increasing the number of dimensions is dramatically shown for the case nr=1. For a false alarm rate of 10^{-2}, the probability of detection increases from about 15% to over 90% as the number of harmonics increases from 1 to 15.

Since the detector which computes q instead of q_k would do even better, there is considerable reason for optimism about the performance of the proposed detector. This optimism is tempered by several factors. First of all, we are not likely to have equal strength independent harmonics; in fact we are not likely to be certain a priori of the number, strength, and dependence of the harmonics. The detector design, specifically the parameter Φ_{Xp}, must be based on some assumption about the signal structure. To an extent, practical receivers might be able to treat Φ_{Xp} as a random variable to be estimated, and try many different combinations of harmonics. At any rate, a receiver designed for a large number of harmonics will perform poorly when trying to detect a signal with only one or two strong harmonics.

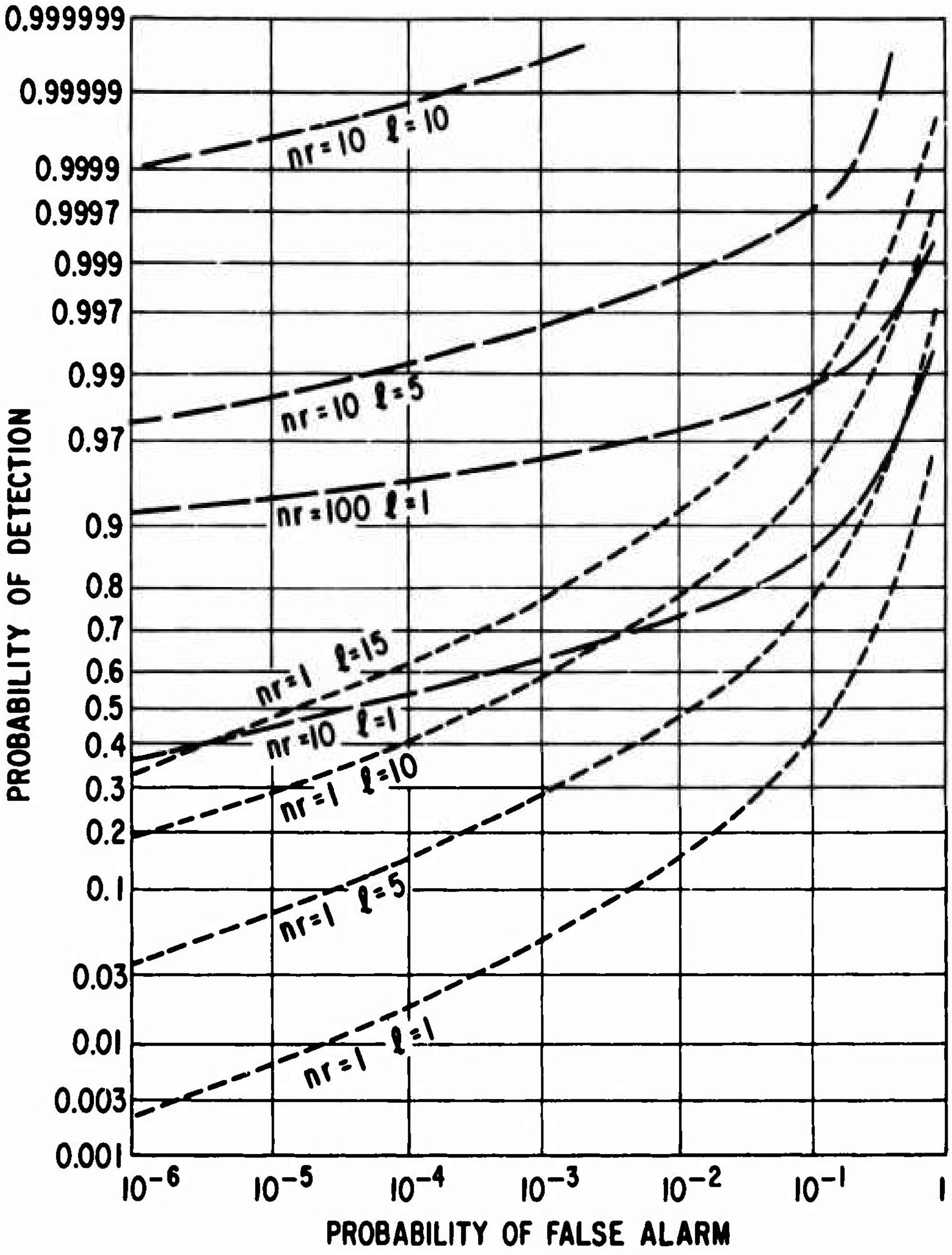

Fig. 3 ROC curves of the equal-strength independent harmonic detector with no incoherent summing and a known period.

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APPENDIX A

We wish to evaluate the integral on the right of Eq. (III-5)

$$\ell(Z|\theta,p) = \int \{dX_i\}_k \, \ell(Z|\theta, \{X_i\}_k), \, f(\{X_i\}_k) \tag{A1}$$

for $\theta = 1$. Substituting Eq. (III-4) for the first term in the integrand, and Eq. (III-10) for the second term gives

$$\ell(Z|\theta=1) = \int \{dX_i\} \, (2\pi)^{-kp/2} |\Phi_{Np}|^{-k/2} \exp\left\{ -\frac{1}{2} \sum_{i=1}^{k} (Z_i - X_i)^* \Phi_{Np}^{-1} (Z_i - X_i) \right\}$$

$$(2\pi)^{-kp/2} (1-\rho^2)^{-p(k-1)/2} |\Phi_{Xp}|^{-k/2}$$

$$\exp\left\{ -\frac{1}{2} (1-\rho^2)^{-1} \left[\sum_{i=1}^{k} X_i^* \Phi_{Xp}^{-1} X_i + \rho^2 \sum_{i=2}^{k-1} X_i^* \Phi_{Xp}^{-1} X_i \right.\right.$$

$$\left.\left. - \rho \sum_{i=1}^{k-1} X_{i+1} \Phi_{Xp}^{-1} X_i - \rho \sum_{i=1}^{k} X_i^* \Phi_{Xp}^{-1} X_{i+1} \right] \right\}$$

$$(2\pi)^{-kp} |\Phi_{Xp}|^{-k/2} |\Phi_{Np}|^{-k/2} (1-\rho^2)^{-p(k-1)/2}$$

$$\int \{dX_i\}_k \exp\left\{ -\frac{1}{2} \left[\sum_{i=1}^{k} (X_i - Z_i)^* \Phi_{Np}^{-1} (X_i - Z_i) \right.\right.$$

$$+ X_1^* (1-\rho^2)^{-1} \Phi_{Xp}^{-1} X_1 + X_k^* (1-\rho^2)^{-1} \Phi_{Xp}^{-1} X$$

$$+ \sum_{i=2}^{k-1} X_i^* (1+\rho^2)(1-\rho^2)^{-1} \Phi_{Xp}^{-1} X_i$$

$$\left.\left. - \sum_{i=1}^{k-1} X_{i+1}^* \rho (1-\rho^2)^{-1} \Phi_{Xp}^{-1} X_i - \sum_{i=1}^{k} X_i^* \rho (1-\rho^2)^{-1} \Phi_{Xp}^{-1} X_{i+1} \right] \right\} . \tag{A2}$$

The strategy is to get the exponent into a sum of quadratic forms in the variables X_i, plus a remainder term. Then each quadratic form can be integrated as a Gaussian joint density function. By investigating the iteration of this process, the portion of the exponent in the brackets can be shown to be equal to

$$\sum_{i=1}^{k} (X_i - A_i B_i)^* A_i^{-1} (X_i - A_i B_i) + Z_i^* \Phi_{Np}^{-1} Z_i - c_i \tag{A3}$$

where

$$\left.\begin{aligned}
A_1^{-1} &= \Phi_{Np}^{-1} + (1-\rho^2)^{-1} \Phi_{Xp}^{-1}, \\
A_i^{-1} &= \Phi_{Np}^{-1} + (1+\rho^2)(1-\rho^2)^{-1} \Phi_{Xp}^{-1} - \rho (1-\rho^2)^{-1} \Phi_{Xp}^{-1} A_{i-1} \rho (1-\rho^2)^{-1} \Phi_{Xp}^{-1} \\
&\qquad \text{for} \quad 1 < i < k, \\
\text{and} \quad A_k^{-1} &= \Phi_{Np}^{-1} + (1-\rho^2)^{-1} \Phi_{Xp}^{-1} - \rho (1-\rho^2)^{-1} \Phi_{Xp}^{-1} A_{k-1} \rho (1-\rho^2)^{-1} \Phi_{Xp}^{-1}
\end{aligned}\right\} \tag{A4}$$

$$\left.\begin{aligned} B_i &= \rho(1-\rho^2)\,\Phi_{Xp}^{-1} X_{i+1} + \sum_{j=1}^{i} \left[\prod_{\ell=1}^{i-j} \rho(1-\rho^2)^{-1}\,\Phi_{Xp}^{-1} A_\ell\right] \Phi_{Np}^{-1}\, Z_j \\ B_k &= \sum_{j=1}^{k} \left[\prod_{\ell=1}^{k-j} \rho(1-\rho^2)^{-1}\,\Phi_{Xp}^{-1} A_\ell\right] \Phi_{Np}^{-1}\, Z_j \end{aligned}\right\} \quad (A5)$$

$$\left.\begin{aligned} c_i &= \left[\sum_{j=1}^{i} \prod_{\ell=1}^{i-j} \rho(1-\rho^2)^{-1}\,\Phi_{Xp}^{-1} A_\ell\, \Phi_{Np}^{-1}\, Z_j\right]^* \\ &\qquad A_i \left[\sum_{j=1}^{i} \prod_{\ell=1}^{i-j} \rho(1-\rho^2)^{-1}\,\Phi_{Xp}^{-1} A_\ell\, \Phi_{Np}^{-1}\, Z_j\right] . \end{aligned}\right\} \quad (A6)$$

The parameters of most interest are the A_i, which are the covariance matrices of the quadratic forms in X_i, and the c_i which determine the ultimate form of the detector.

Now carrying out the integration indicated in Eq. (A2) gives

$$\begin{aligned} \ell(Z\,|\,\rho=1) &= (2\pi)^{-kp/2}\, |\Phi_{Xp}|^{-k/2}\, |\Phi_{Np}|^{-k/2} (1-\rho^2)^{-p(k-1)/2} \prod_{i=1}^{k} |A_i|^{1/2} \\ &\quad \exp\left\{-\frac{1}{2}\sum_{i=1}^{k} Z_i^*\, \Phi_{Np}^{-1}\, Z_i + \frac{1}{2}\sum_{i=1}^{k} c_i\right\} . \end{aligned} \qquad (A7)$$

APPENDIX B

Since the coefficients A_i and c_i in Eq. (III-16) are too complex to be implemented easily, some simplifying assumptions are necessary. We shall investigate the case of low signal-to-noise ratio. We define low signal-to-noise ratio to be the condition that the eigenvalues of the signal-to-noise ratio matrix $\Phi_{Xp}\Phi_{Np}^{-1}$ are less than unity. Now we note that for low signal-to-noise ratio

$$
\begin{aligned}
A_1 &= \left[\Phi_{Np}^{-1} + (1-\rho^2)\,\Phi_{Xp}^{-1}\right]^{-1} \\
&= \left[\Phi_{Xp}\Phi_{Np}^{-1} + (1-\rho^2)^{-1} I\right]^{-1} \Phi_{Xp} \qquad \text{(B1)} \\
&\sim (1-\rho^2)\,\Phi_{Xp}\ .
\end{aligned}
$$

Substituting this result into Eq. (B4) for A_2, A_3, etc., we find that in general

$$
A_i \simeq \begin{cases} (1-\rho^2)\,\Phi_{Xp}; & i < k \\ \Phi_{Xp} & ; i = k\ . \end{cases} \qquad \text{(B2)}
$$

Substituting this into Eq. (B6) for c_i gives

$$
c_i \simeq \begin{cases} \left(\sum_{j=1}^{i} \rho^{i-j}\,\Phi_{Np}^{-1} Z_j\right)^{*} (1-\rho^2)\,\Phi_{Xp}\left(\sum_{j=1}^{i} \rho^{i-j}\,\Phi_{Np}^{-1} Z_j\right); & i < k \\ \left(\sum_{j=1}^{k} \rho^{k-j}\,\Phi_{Np}^{-1} Z_j\right)^{*} \Phi_{Xp}\left(\sum_{j=1}^{k} \rho^{k-j}\,\Phi_{Np}^{-1} Z_j\right) & ; i = k\ . \end{cases} \qquad \text{(B3)}
$$

Note that the last term in the sum of the c_i is emphasized much more strongly than the other terms.

APPENDIX C

The response of the weighted circulating adder to a unit impulse at time $t = 0$ is

$$y(t) = \sum_{j=0}^{\infty} \rho^{j} \delta(t-jp)$$

$$= u_{-1}(t) \rho^{t/p} \sum_{j=-\infty}^{\infty} \delta(t-kp) \tag{C 1}$$

where p is the length of the waveforms to be added, and $u_{-1}(t)$ is the unit step function.

$$u_1(t) = \begin{cases} 0 \text{ for } t < 0 \\ 1 \text{ for } t \geq 0 \end{cases} .$$

The filter frequency response is the Fourier transform of the system response to a unit impulse applied at time $t = 0$. Taking the Fourier transform of both sides of Eq. (C 1) gives

$$Y(f) = \frac{1}{(1/p)\ln(1/\rho)+j2\pi f} * \sum_{h=-\infty}^{\infty} (1/p)\, \delta(f-h/p)$$

$$= \sum_{n=-\infty}^{\infty} \frac{(1/p)}{(1/p)\ln(1/\rho)+j2\pi(f-h/p)} \tag{C 2}$$

where * denotes convolution here. This is the system function of a comb filter with "teeth" having the form of the functions on the right of Eq. (C 2). The teeth are spaced $1/p$ apart. The tooth width, as measured between the $1/2$ power points, is $(1/\pi p) \ln (1/\rho)$.

In Section IV-A-3, it is argued that it is desirable to know how far apart values of p should be chosen so that the comb filter would be sure to pass significant energy from a nearly periodic signal with period, say, p_0. Since varying the value of p makes the largest change in the center frequency tooth corresponding to the highest harmonic h_{max}, we need only concentrate on that tooth. We shall adopt the criterion that the increment Δp should not move this tooth further than the distance between its half power points. This tooth is centered at the frequency

$$f = \frac{h_{max}}{p} . \tag{C 4}$$

Therefore,

$$f+\Delta f = \frac{h_{max}}{p+\Delta p} \tag{C 5}$$

and

$$\Delta f = \frac{h_{max}}{p+\Delta p} - \frac{h_{max}}{p}$$

$$\simeq \frac{h_{max}}{p^2} \Delta p \text{ for } \Delta p \ll p . \tag{C 6}$$

Substituting the distance between the half-power points for Δf into Eq. (C 6)

$$\Delta p = \frac{p}{\pi h_{max}} \ln \left(\frac{1}{\rho}\right) . \tag{C 7}$$

We also want to know how far apart the values of ρ should be chosen so that the value of ρ which gives the maximum processor output will not be missed. The main effect of changing ρ is to change the width of the teeth of the comb filters. If the teeth are wider than the line width of the nearly periodic signal, too much noise will get through. If the teeth are too narrow, only a portion of the signal will be passed by the comb teeth. This latter effect tends to spread the signal energy over the comb filters for several different values of p. It seems that a reasonable criterion is to require that the changes in ρ increment the comb tooth width by a factor of two, so that the signal-to-noise ratio does not change by more than a factor of two. Therefore,

$$\frac{1}{\pi p} \ln \frac{1}{\rho - \Delta\rho} = \frac{2}{\pi p} \ln \frac{1}{\rho} . \qquad \text{(C8)}$$

Solving this equation for $\Delta\rho$ gives

$$\Delta\rho = \rho(1-\rho) . \qquad \text{(C9)}$$

APPENDIX D

We wish to find the probability density function of the random variable q_k given by Eq. (IV-3) when the noise is white and the waveforms of the periods of the nearly periodic signal are of the form specified by Eq. (IV-14).

We note that Eq. (IV-10) can be written as follows:

$$X_p(t) = \mathrm{Re} \sum_m a_m \exp(j\alpha_m) \exp[j2\pi(m/p)t]$$

$$= \sum_m a_{mr} \cos 2\pi(m/p)t - a_{mi} \sin 2\pi(m/p)t \tag{D1}$$

where

$$a_{mr} = a_m \cos \alpha_m$$

$$a_{mi} = a_k \sin \alpha_k \ .$$

Then by the assumptions made on the a_n and α_n, the following statistical averages can be computed

$$\langle a_{mr}^2 \rangle = \langle a_{mi}^2 \rangle = \frac{1}{2} \langle a_m^2 \rangle = \sigma_X^2 \tag{D2}$$

$$\langle a_{mr} a_{nr} \rangle = \langle a_{mi} a_{ni} \rangle = 0 \ ; \quad m \neq n \tag{D3}$$

$$\langle a_{mr} a_{ni} \rangle = 0 \ ; \ \text{all } m,n \ . \tag{D4}$$

Furthermore, the coefficients a_{mr} and a_{mi} are Gaussian random variables since they are linear transformations of the time samples of the sample functions of a Gaussian process.

The equations (D1) through (D4) show that the set of functions $\{\cos 2\pi(m/p)t; \sin 2\pi(m/p)t\}$ is an (orthonormal) basis, with statistically independent coefficients, for the sample functions in each period of the nearly periodic waveform. Note that the cosine or sine coefficients of the same order in different periods cannot be independent because of the assumed period to period correlation ρ. Also, note that all the coefficients a_{mr} and a_{mi} have the same variance σ_X^2.

In this new coordinate system of harmonic cosines and sines, the covariance matrices Φ_{Np} and Φ_{Xp} are diagonal. The Φ_{Np} matrix is diagonal because white noise has independent coefficients in any basis.

$$\Phi_{Np} = 2\ell \sigma_N^2 I \ . \tag{D5}$$

The Φ_{Xp} matrix is diagonal, and will be of the form

$$\Phi_{Xp} = 2\ell \sigma_X^2 I \ . \tag{D6}$$

With the noise and covariance matrices given by Eqs. (D5) and (D6), the expression for q_k given by Eq. (IV-3) becomes

$$q_k = \sum_{i=1}^{\ell} \sigma_{Xi}^2 \left(\sum_{j=1}^{k} \rho^{k-j} \sigma_N^{-2} z_{ij} \right)^2$$

$$= \sum_{i=1}^{\ell} \left(\sigma_{Xi} \sigma_N^{-2} \sum_{j=1}^{k} \rho^{k-j} z_{ij} \right)^2 \tag{D7}$$

$$= \sum_{i=1}^{\ell} y_i^2 \ .$$

where the z_{ij} are the entries of the matrix Z_i.

Now, since q_k is the sum of squares of identically distributed, zero mean, Gaussian random variables, it is chi squared distributed, with 2ℓ degrees of freedom. Therefore, the only parameters needed to completely specify their distribution are ℓ and $\langle y_i^2 \rangle$. We note that since

$$\langle z_{ij_1} z_{ij_2} \rangle = \delta_{j_1 j_2} \sigma_N^2 + \theta \rho^{|j_2 - j_1|} \sigma_X^2 \tag{D8}$$

then

$$\begin{aligned} \langle y_i^2 \rangle &= \sigma_X^2 \sigma_N^{-4} \sum_{j_1=1}^{k} \sum_{j_2=1}^{k} \rho^{k-j_1} \rho^{k-j_2} \langle z_{ij_1} z_{ij_2} \rangle \\ &= \sigma_X^2 \sigma_N^{-4} \left[\sum_{j=1}^{k} \rho^{2(k-j)} \sigma_N^2 + \theta \sum_{j_1=1}^{k} \sum_{j_2=1}^{k} \rho^{k-j_1} \rho^{k-j_2} \rho^{|j_2-j_1|} \sigma_X^2 \right] . \end{aligned} \tag{D9}$$

We assume now that k is much larger than $(1-\rho^2)^{-1}$, i.e., k is sufficiently large that the correlation between the first and the kth period is essentially zero. Then the following approximations are valid

$$\sum_{j=1}^{k} \rho^{2(k-j)} \simeq \frac{1}{1-\rho^2} \tag{D10}$$

$$\sum_{j_1=1}^{k} \sum_{j_2=1}^{k} \rho^{k-j_1} \rho^{k-j_2} \rho^{|j_2-j_1|} \simeq \frac{1}{1-\rho^2} \left(\frac{1+\rho^2}{1-\rho^2} \right) . \tag{D11}$$

Substituting Eqs. (D10, 11) into Eq. (D9)

$$\langle y_i^2 \rangle = \frac{r}{1-\rho^2} \left(1 + \theta \, \frac{1+\rho^2}{1-\rho^2} \, r \right) = \sigma_\theta^2 \tag{D12}$$

where $r = \sigma_X^2 \sigma_N^{-2}$, the signal-to-noise ratio per dimension, which is the same as the over-all ratio.

The tables of the chi-squared distribution are given for $\sigma_\theta^2 = 1$.(31) Thus by dividing the variable q_i by σ_θ^2, the probabilities of detection and false alarm can be found. When the threshold is set at some level η, the probabilities of detection is the probability that q_k/σ_1^2 exceeds η/σ_1^2. The probability of false alarm is the probability q_k/σ_0^2 exceeds η/σ_0^2. Since the tables give the probability that the variable exceeds chi squared for various values of chi squared and ℓ, the probabilities of detection and false alarm can be read directly.

On making up the ROC's, it is desirable to make the number of lines one of the parameters. It must be remembered that each line consists of two dimensions. Furthermore, ρ as a parameter is hard to measure directly. A more easily measured parameter is the number of periods n at which the correlation between the first and the nth period drops to 1/2, i.e., where $\rho^n = 1/2$. A further simplification results if we assume $n > 10$, i.e., $\rho > 0.933$, in which case $1+\rho^2 \simeq 2$.

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13 ABSTRACT

This report proposes a design of an adaptive receiver for the detection and estimation of nearly periodic signals in additive Gaussian noise. A nearly periodic signal is defined to be a sample function of a Gaussian random process which can be divided into equal length intervals, called periods, in such a manner that the correlation between periods decreases exponentially with their separation. The receiver computes a low signal-to-noise ratio conditional likelihood ratio from which the observer must make decisions. The likelihood ratio is conditional because the receiver estimates any unknown parameters necessary for the computation of the true likelihood ratio. Thus the receiver can only compute a likelihood ratio conditioned upon these estimates being the true values of the unknown parameters. The receiver consists of pre-emphasis filters followed by a comb filter, energy detector, and weighted summation. A theoretical evaluation of the receiver, in terms of ROC curves, is made for the special case of nearly periodic signals with statistically independent equal-strength harmonics in white noise of known power.

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